Analog Computer Applications
Application Note #2
Undergraduate Engineering Physics Laser-Themed STEM Project

Mode-Locked Gaussian Laser Power Pulse Simulation

1 Introduction

This application simulates a mode-locked Gaussian laser power pulse without the use of an antilogarithmic computing element.

The first task is to produce a differential equation from a typical Gaussian function. This 'reverse engineering' approach is discussed in detail below.

2 Mathematical modeling

Assume a time-shifted mode-locked Gaussian laser power pulse:

$$P = P_{max}e\hat{}(-4\ln(2)((t - \tau_s)/\tau_p)^2) \qquad (1)$$

where

P = laser power
P_{max} = maximum (or peak) laser power
t = time
τ_s = time shift
τ_p = FWHM (Full Width at Half Maximum)

Letting $\gamma = 4\ln(2)/\tau_p^2$,

$$P = P_{max}e\hat{}(-\gamma(t - \tau_s)^2)$$

Differentiating P with respect to t,

$$\dot{P} = -2\gamma(t - t')P_{max}e\hat{}(-\gamma(t - \tau_s)^2$$

$$\dot{P} = -2\gamma(t - \tau_s)$$

$$\dot{P} + 2\gamma(t - \tau_s)P = 0 \text{ with } P(0) = P_0 \quad (2)$$

where

$\dot{P} = dP/dt$
$\tau_s =$ time shift
$P_0 =$ laser power at initiation of program run

Equation (2) is the desired differential equation for which the analog computer is well suited to solve using a few operational amplifiers and an analog multiplier.

Letting $e = 2\gamma(t - \tau_s)$, (3)

$$\dot{P} + eP = 0 \text{ with } P(0) = P_0 \quad (4)$$

Generating e: The operational amplifier (op-amp) integrator to the rescue!

For an op-amp integrator, with a constant input voltage,

$V_{out} = -(V_{in}/\tau)\,t + V_0 = -(V_{in}/\tau)(t - (V_{in}/V_0)\tau)$

where

$V_{in} =$ input voltage
$V_0 =$ initial voltage
$\tau = RC$ (time constant)

Letting $V_{out} = e$, $V_{in} = \alpha$, and $V_0 = \beta$,

$$e = -(\alpha/\tau)(t - (\beta/\alpha)\tau \quad (5)$$

Comparing (5) to (3),

$\alpha = 2\gamma\tau = 8\ln(2)\tau/\tau_p^2$ and $\beta = \alpha\tau_s/\tau = 2\gamma\tau\tau_s/\tau = 2\gamma\tau_s = 8\ln(2)\tau_s/\tau_p^2$

Letting $P_0 = 0.100$, $\tau = 1.760$, $\tau_p = 8.331$, and $\tau_s = 7.500$,

$\alpha = 0.141$ and $\beta = 0.599$

3 Computer setup

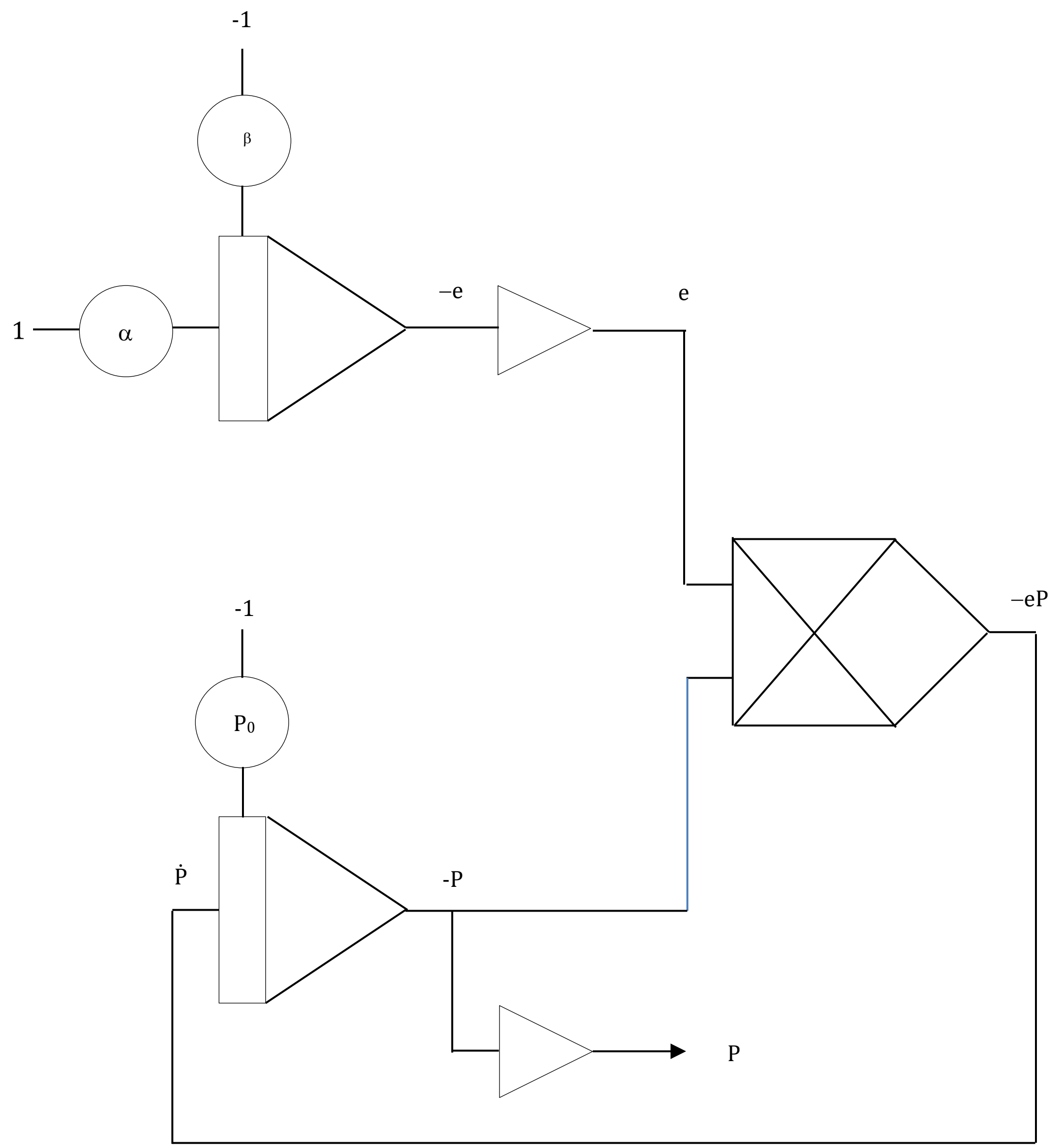

Figure 1: Computer setup for mode-locked Gaussian laser power pulse simulation

Parameter	Value
α	0.141
β	0.599
P_o	0.100

Table 1: Parameter settings for mode-locked Gaussian laser power pulse simulation

Result

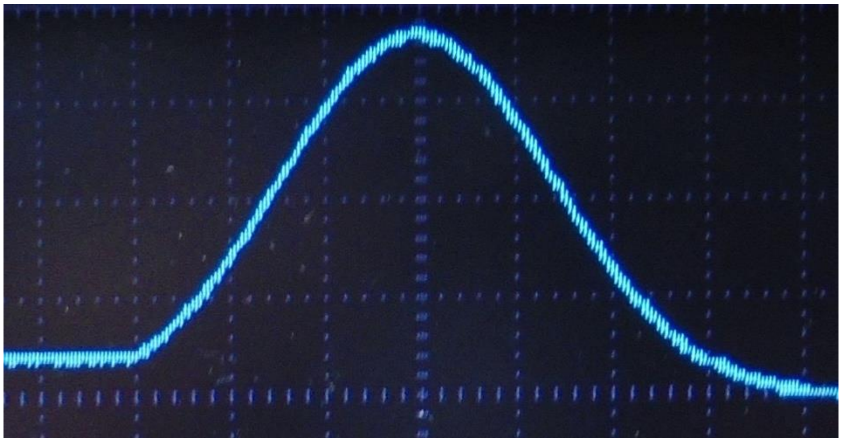

Figure 2: Mode-locked Gaussian laser power pulse*

*For this application note, the display was produced during a single run by a differential equation analog computer prototype using discrete components with tolerances between 1% and 10%.