

Simulating the flight of a glider

1 Introduction

In the early years of the 20th century, the English engineer and polymath FREDERICK WILLIAM LANCHESTER¹ discovered the phenomenon of *phugoid oscillation*, which describes the motion of an aircraft pitching up and climbing only to pitch down and descend.² The *angle of attack*, AOA, of the airplane is next to constant during motion while it experiences an oscillatory exchange of airspeed and altitude. A detailed description of this phenomenon can be found in [SIMANCA et al. 2002, pp. 3:1 ff.] on which the following derivation is based.

Figure 1³ shows the basic glider airplane considered in the following. φ is the angle between the centerline of the glider and the horizontal axis, while drag and lift are proportional to the glider's velocity v . Introducing the drag coefficient R , the drag will be Rv^2 while lift will be considered to be just equal to v^2 in the following.

This yields

$$m\dot{v} = -mg \sin(\varphi) - Rv^2 \quad (1)$$

which is obviously independent of the glider's mass m . To get another equation for the φ , the centripetal force

$$F_z = \frac{mv^2}{r}$$

is used. With

$$\dot{\varphi} = \frac{v}{r}$$

this can be rewritten as

$$F_z = mv\dot{\varphi},$$

¹23.10.1868 – 08.03.1946

²See [LANCHESTER 1908].

³Cf. [SIMANCA et al. 2002, p. 3:2].

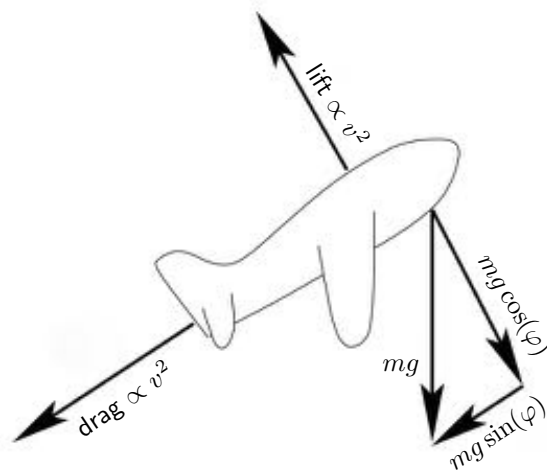


Figure 1: Basic glider airplane

which must be equal to $v^2 - mg \cos(\varphi)$ with v^2 being equal to the lift yielding

$$mv\dot{\varphi} = v^2 - mg \cos(\varphi), \quad (2)$$

which, too, does not depend on m . Setting $g := 1$ to simplify things further, finally yields the following two equations based on (1) and (2):

$$\begin{aligned} \dot{v} &= -\sin(\varphi) - Rv^2 \\ \dot{\varphi} &= \frac{v^2 - \cos(\varphi)}{v} \end{aligned} \quad (3)$$

(3) is better written as

$$\dot{\varphi} = v - \frac{\cos(\varphi)}{v}$$

for implementation on an analog computer.

To display the path of flight of this glider, the required x, y -coordinate tuple can



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be generated by integrating over

$$\dot{x} = v \cos(\varphi) \text{ and}$$

$$\dot{y} = v \sin(\varphi).$$

The free parameters of this simulation are listed in the following:

$v(0)$: Initial velocity of the glider. This, basically, is the initial value of the integrator yielding $-v$.

R : The drag coefficient of the glider.

$\varphi(0)$: Initial angle of attack. Since $\sin(\varphi)$ and $\cos(\varphi)$ are derived based on $\dot{\varphi}$ instead of φ since φ is not easily restricted to a fixed interval, setting $\varphi(0)$ requires the initial conditions of two integrators to be set to $\cos(\varphi(0))$ and $\sin(\varphi(0))$, respectively.

To make a (very long) story short, scaling this problem is pretty involved and may be left to the interested reader.⁴

2 Implementation

This problem was solved on a historic machine, namely a Telefunken RA770.⁵ Since this machine uses quarter square multipliers, all multipliers need their input values with both positive and negative signs which clutters the schematic a bit and deviates from modern multipliers. Some multipliers (those in the expansion chassis for non-linear computing elements, require a special buffer amplifier which needs a feedback of 10 instead of 1, which is pretty unusual from today's perspective.

The first subcircuit is the generation of $\pm \sin(\varphi)$ and $\cos(\varphi)$ based on $\dot{\varphi}$ as input as shown in figure 2. There is nothing special about this circuit – since the simulation

⁴This is the revenge for all the books I read during my university time which usually left unpleasant tasks like this to the reader. . . :-)

⁵This particular model is without much debate the top model of Telefunken's analog computer family and was introduced in 1966 and built until 1975. It was often part of a hybrid computer installation. Only a few of these machines are known to have survived.

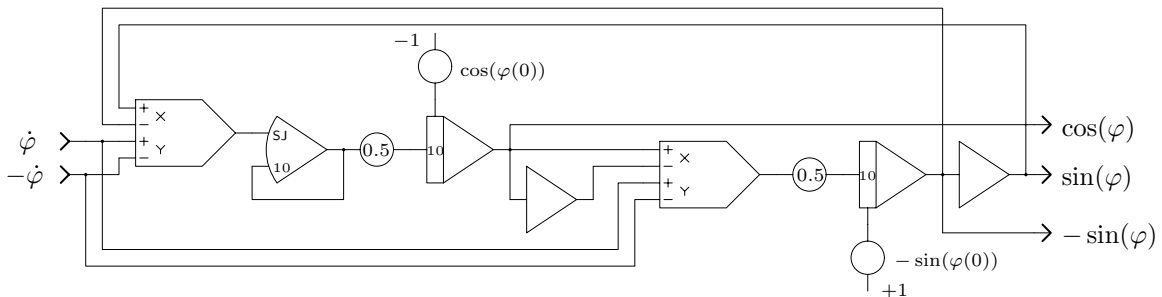


Figure 2: Sine/cosine circuit

will typically be run in repetitive mode with rather high values of k_0 , no amplitude stabilization is required here.

Figure 3 shows the partial computer setup yielding $\pm v$ and v^2 . The two coefficient potentiometers for R and the scaling factor 0.5 could be combined into a single potentiometer but this allows R to be set directly without having to scale its values manually before setting. Note the input weights of 10 in this subcircuit. These were determined by manual scaling and yields a greater sensitivity of the glider to R and the gravitational pull.

Deriving $\pm \dot{\varphi}$ requires a divider. Typically, division on an analog computer should be abhorred due to two reasons: Scaling gets pretty complicated quickly with division circuits, and division circuits implementing the inverse function to multiplication by means of an open amplifier tend to oscillate and typically require a small external capacitor to prevent such oscillations. In this case, the division could not be avoided. The actual implementation of the division circuit on the RA770 is pretty involved and not shown in figure 4 where only the abstract symbol for a divider is used.

Deriving the x and y coordinates of the glider is straight-forward as shown in figure 5 since v , $\sin(\varphi)$, and $\cos(\varphi)$ are known ahead.

Figure 6 shows the actual setup on a Telefunken RA770. The black rectangular boxes are just boxes with six interconnected jacks as it is not possible to stack the special Telefunken patch cables like banana plugs. All in all this simulation requires

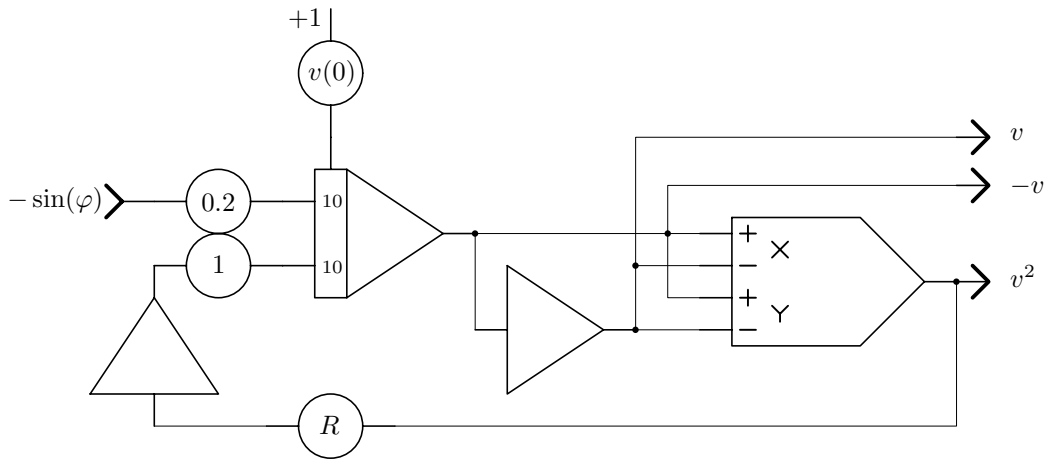


Figure 3: Computing $\pm v$ and v^2 , the scale factor 1 on the lower input of the integrator should be $\frac{1}{2}$ if scaled "by the book", but setting this parameter to 1 yields more sensitivity with respect to R

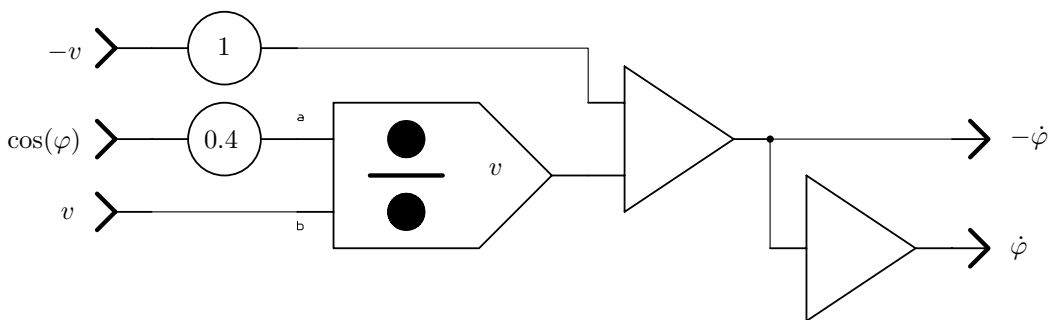


Figure 4: Computing $\pm\dot{\varphi}$

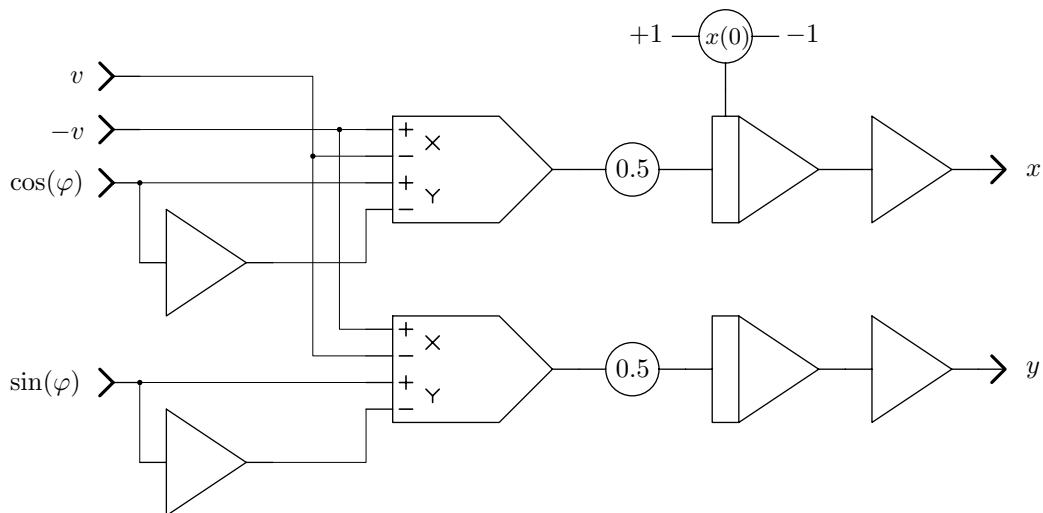


Figure 5: Computing x and y of the glider

seven summers, five integrators, 12 potentiometers,⁶ one “free” potentiometer, five multipliers, and one divider.⁷

3 Results

It is quite fun to play with the two free parameters $v(0)$ and R . Figure 7 shows a typical simulation result. The simulation was run at high speed in repetitive mode so that a flicker free oscilloscope display could be achieved. This particular result was obtained with the following settings for the free parameters:

⁶Two of those could be eliminated as they have been set to 1 during the scaling process.

⁷Please note that the inverters required by the quarter square multipliers of the RA770 have not been included in that list since these can be seen as parts of the amplifiers.

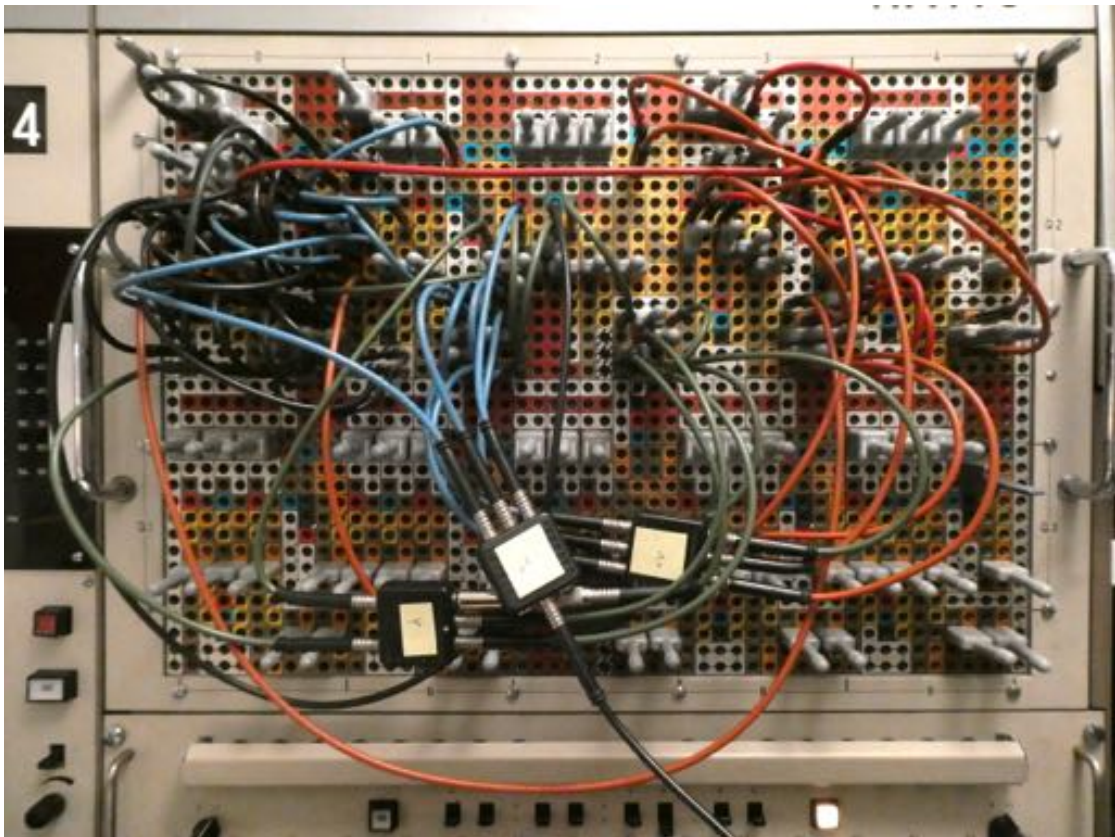


Figure 6: Problem setup on a Telefunken RA770

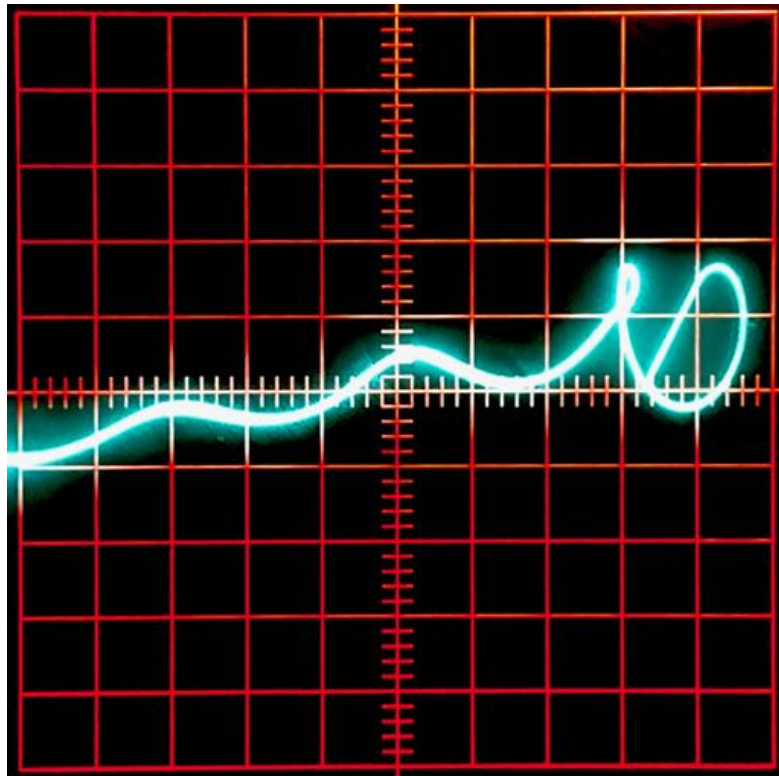


Figure 7: typical simulation result

Parameter	Value
$-\sin(\varphi(0))$	0.707
$\cos(\varphi(0))$	0.707
$v(0)$	0.79
R	0.04
$-g$	0.107

The glider starts at $y = 0$ at the right and is thrown at a rather steep angle to the right. As v is pretty small, it goes into a dive, turns its direction to the left and gains



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velocity and thus lift which yields height. It makes a tiny loop before it slowly wiggles down.

References

- [LANCHESTER 1908] FREDERICK WILLIAM LANCHESTER, *Aerial Flight: Aerodnetics*, London, Constable, 1908
- [SIMANCA et al. 2002] SANTIAGO R. SIMANCA, SCOTT SUTHERLAND, *Notes for MAT 331 – Mathematical Problem Solving with Computers*, The University at Stony Brook, <https://www.math.stonybrook.edu/~scott/Book331/331book.pdf>